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Memory response in plane wave reflection in generalized magneto-thermoelasticity

Nihar Sarkar[a], Soumen De [b] and Nantu Sarkar [b]

[a]Department of Mathematics, City College, Kolkata, India; [b]Department of Applied Mathematics, University of Calcutta, Kolkata, India

ABSTRACT
This paper demonstrates the reflection phenomenon of the mag neto-thermoelastic plane waves from a stress-free surface of a homogeneous, isotropic, thermally and electrically conducting solid half-space. We employ the generalized thermoelasticity model with memory-dependent derivative (MDD) for this study. We find that three basic plane waves consisting of two sets of coupled longitudinal waves and one independent shear type wave may travel with distinct speeds in the medium. The speeds of the coupled longitudinal waves are plotted graphically to predict a comparison for the MDD and Lord–Shulman models. At last, for an appropriate material, the reflection coefficients are computed numerically and presented graphically with the angle of incidence of the incident (vertically polarized) shear type wave and the obtained results are explained.

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Memory-dependent derivative; time-delay; kernel function; *P*- and *SV* -waves; dispersion; reflection

1. Introduction

Non-local continuum field theories are concerned with the physics of material bodies whose behavior at a material point is influenced by the state of all points of the body [1]. The non-local theory generalizes the classical field theory in two respects: (1) the energy balance law is considered valid globally; and (2) the state of the body at a material point is described by the response functional. In this description, non-locality in time is known as memory dependence. The theory of heat conduction in continuous media with memory has drawn the attention of many researchers. Initially, the motivation was to avoid the unpleasant feature of the classical coupled heat conduction model [2] in which the the thermal signals propagate with infinite speed. To overcome this technical issue, by incorporating the thermal relaxation times into the heat flux model, several researchers have attempt to modify the coupled dynamic thermoelasticity theory. These studies were based on several modifications of Fourier's law of heat conduction. The aim of enhancement was to derive hyperbolic-type partial differential equations to govern the heat conduction properties to simultaneously satisfy the following conditions:

CONTACT Nantu Sarkar nsarkarindian@gmail.com Department of Applied Mathematics, University of Calcutta, 700009 Kolkata, India

(1) finiteness of heat signal propagation speed, (2) spatial propagation of thermoelastic waves without attenuation, and (3) existence of distortionless waveforms akin to the classical d'Alembert-type waves. Cattaneo [3] proposed a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. Lord and Shulman [4], Green and Lindsay [5], and Green and Naghdi [6] proposed three different generalized heat conduction equations which are the most discussed hyperbolic-type heat equations in the literature. These models keep running under the mark of "hyperbolic thermoelasticity".

Wave reflection phenomena may applicable in various fields like geophysical exploration, seismology, engineering etc. Several problems on reflection of plane harmonic wave in coupled and generalized thermoelasticity theories have been investigated by many authors [7]. Great attention has been employed inside the nuclear reactors to influence its design as well as functions and consequently in the study of magneto-thermoelastic interactions using the linear theory of generalized thermoelasticity. The interplay of Maxwell electromagnetic field with the motion of deformable solid is largely being undertaken by many authors due to its possible applications to geophysical problems and certain other topics in acoustics. The earth is subjected to its own magnetic field and the material of the earth may be assumed as electrically conducting medium. Thus the magneto-elastic nature of the earth's material may affect the propagation of plane waves [8]. Paria reported magneto-thermo-elastic plane waves in [9]. Nayfeh and Nemat-Nasser [10] reported electromagneto-thermoelastic plane waves in solids with thermal relaxation and Agarwal [11] studied a problem on electromagneto-thermoelastic plane waves. Roychoudhuri [12] studied electro-magneto-thermo-elastic plane waves in rotating media with thermal relaxation. Abd-Alla et al. [13] reported a work on the reflection of the generalized magneto-thermo-viscoelastic plane waves. Roy Choudhuri and Banerjee [14] investigated magneto-viscoelastic plane waves in rotating media in the generalized thermoelasticity II. Some other notable works can be found in the literatures [15–20].

Memory-dependent derivatives (MDDs) were first incorporated in Fourier's law of heat conduction, a new hyperbolic-type heat conduction equation by Wang and Li [21]. This new generalization of hyperbolic-type heat conduction models is accepted as the modified heat conduction law with measuring memory. In this paper, we investigate the reflection of the magneto-thermoelastic plane waves from a stress-free and thermally insulated surface of a homogeneous, isotropic, thermally and electrically conducting solid half-space in the frame of the generalized thermoelasticity under heat transfer with an MDD [22]. We find that three basic plane waves consisting of two sets of coupled longitudinal waves and one independent shear type wave may travel with distinct phase speeds in the medium considered. All these waves are influenced by the presence of magnetic field and the MDD. The formulae for various reflection coefficients are determined for an incident SV-type wave at a thermally insulated stress-free boundary. The phase speeds of the coupled longitudinal waves are plotted graphically to predict a comparison between the MDD and the Lord–Shulman (LS) models. The numerical results for the reflection coefficients for various values of the angle of incidence of the incident (vertically polarized) shear wave are illustrated graphically for copper like material and highlight the effect of the magnetic pressure number, time-delay, Poison ration, and the thermoelastic coupling parameter.

2. Memory in material modeling

Scott Blair's model [23], which is basically a material model, includes a formula for memory phenomena in various disciplines. The model takes the form

$$ {}^{0}\mathcal{D}_t^{\alpha}\epsilon(t) = \kappa\sigma(t), \tag{1} $$

where ${}^{0}\mathcal{D}_t^{\alpha}\epsilon(t)$ denotes the fractional-order derivative which depends on the strain history from 0 to t. For integral value of $\alpha = n$, ${}^{0}\mathcal{D}_t^{\alpha}\epsilon(t) = d^n\epsilon(t)/\mathrm{d}t^n$, and $\kappa > 0$ is a constant.

A fractional-order derivative is a generalization of an integer order derivative and integral. It originated from a letter of L'Hopital to Leibnitz in 1695 regarding the meaning of the half-order derivative. It is a promising tool for describing memory phenomena [24, 25]. The kernel function of a fractional derivative is termed the memory function, but it does not replicate any physical process. Imprecise physical meaning has been a big obstacle that keeps fractional derivatives lagging far behind the integer-order calculus. There are several definitions of a fractional derivative. The Riemann–Liouville derivative is one of the most standard definitions

$$ {}^{0}\mathcal{D}_t^{\alpha}\epsilon(t) = \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{\mathrm{d}t^n}\int_0^t \frac{\epsilon(s)}{(t-s)^{1+\alpha-n}}\mathrm{d}s, \quad m-1 \le \alpha < m, $$

where $\Gamma(\cdot)$ is the Euler's gamma function and m is an integer. A memory process generally consists of two stages: the first is short, with permanent retention at the beginning, and it cannot be neglected in general, and the second is governed by the fractional model Equation (1). The critical point between the fresh stage and the working stage is usually not the origin. This is quite different from the traditional fractional models of one stage. The key point is that the order of a fractional derivative is an index of memory. The dimensionless form of the solution of Equation (1) is

$$ E(\eta) = \eta^{\alpha} - (\eta - 1)^{\alpha}, \tag{2} $$

where $\eta = t/t_M$ and $E(\eta) = \epsilon(t)/\epsilon_M$, where ϵ_M is the strain at the end of time of creeping $t = t_M$. Equation (2) reveals that $E(\eta)$ increases with an increase in α. The higher the value of the index α, the slower is the forgetting during the process. In particular, at $\alpha = 0$, $E = 0$, meaning that "nothing is memorized", and $E=1$ for $\alpha = 1$ which means that "nothing is forgotten". Therefore, the fractional order α is basically termed as the index of the memory effect.

For a standard creep and recovery process, the specimen is usually loaded under a constant stress $\sigma(t) = \sigma_0$ from 0 to t_M, and the load is removed at the instant $t = t_M$, then $\sigma(t) = 0$ for $t \ge t_M$. If $H(t)$ is the Heaviside function, Equation (1) takes the following form:

$$ {}^{0}\mathcal{D}_t^{\alpha}\epsilon(t) = \kappa\sigma_0\left(H(t) - H(t-t_M)\right), $$

where ${}^{0}\mathcal{D}_t^{\alpha}\epsilon(t)$ is the Riemann–Liouville fractional-order derivative with zero initial condition. The superposition method gives the solution of the above equation as follows:

$$ \epsilon(t) = \frac{\kappa\sigma_0}{\Gamma(1+\alpha)}\left[t^{\alpha}H(t) - (t-t_M)^{\alpha}H(t-t_M)\right]. $$

This is in agreement with the early observation of the behavior of some viscoelastic materials.

Equation (1) works not only in modeling viscoelastic materials, but also in modeling biological kinetics with memory. For example, for protein adsorption kinetics, if the symbols σ and ϵ are replaced with the concentration c and the surface density ϱ of fibronectin therein, respectively, then

$$\sigma(t) = \sigma_0\,[H(t) - H(t-t_M) + H(t-t_N)]\,,$$

where $\sigma_0 = 50\,\mu\text{g/ml}$, $t_m = 240\,s$, and $t_N = 1150\,\text{s}$. The absorbed density is found to be

$$\epsilon(t) = \frac{\kappa\sigma_0}{\Gamma(1+\alpha)}\left[t^{\alpha}H(t) - (t-t_M)\alpha H(t-t_M) + (t-t_N)\alpha H(t-t_N)\right].$$

3. Memory-dependent derivatives

In the last decade, non-integral (fractional)-order derivatives and fractional differential equations have gained considerably more attention in the fields of applied sciences and various engineering disciplines [22, 26, 27]. Gorenflo and Mainardi [28], and Atanackovic et al. [29] provided diverse theoretical advances and recent applications of fractional calculus. One hindrance to the wider use of fractional-order methods by engineers is the absence of a simple geometric picture for the fractional-order integral. There are several definitions of fractional derivatives (e.g. Riemann–Liouville, Caputo, Reisz, and Grunwald-Letnikov [28]), each of which has specific advantages and limitations, particularly when used to define a distribution of fluxes into a control volume or the effects of fading memory on the forces applied in a free body diagram. Diethelm [30] incorporated a kernel function and modified a Caputo-type fractional-order derivative as

$$\mathcal{D}_a^{\alpha} f(t) = \int_a^t k_\alpha(t-\xi) f^m(\xi)\mathrm{d}\xi,$$

where $k_\alpha(t-\xi)$ is the kernel function, and f^m is the mth order derivative. In applications, $k_\alpha(t-\xi)$ takes some specific form, e.g.

$$k_\alpha(t-\xi) = \frac{(t-\xi)^{m-\alpha-1}}{\Gamma(m-\alpha)}.$$

Wang and Li [21] proposed another form of the fractional derivative with arbitrary kernel $K(t-\xi)$ (can be chosen freely) over a slipping interval $[t-\tau, t]$ as follows:

$$\mathcal{D}_\tau^{(1)} f(t) = \frac{1}{\tau}\int_{t-\tau}^t K(t-\xi) f'(\xi)\mathrm{d}\xi,$$

where τ (> 0) is called the delay time, which also can be chosen freely. The preceding modifications of fractional-ordered derivatives are termed MDDs. In general, the mth order MDD of a differentiable function $f(t)$ relative to the time delay $a > 0$ is defined as

$$\mathcal{D}_a^{(m)} f(t) = \frac{1}{\tau}\int_{t-a}^t K(t,\xi) f^{(m)}(\xi)\mathrm{d}\xi,$$

where the time delay a denotes the memory scale, and the kernel function $K(t.\xi)$ must be a differentiable function with respect to its arguments. The kernel function and the memory scales must be chosen in such a way that they are compatible with the physical problem, so this type of derivative provides more possibilities to capture the material response. Generally, the memory effect needs weight $0 \leq K(t-\xi) \leq 1$ for $\xi \in [t-\tau, t]$ so that the magnitude of $D_\tau f(t)$ is usually smaller than that of the common derivative $f'(t)$. Simply the right-hand side of (1) is a weighted mean of $f'(t)$. As $\xi \in [t-\tau, t]$, one can easily understand that the function $f(\xi)$ takes value from different points on the time interval $[t-\tau, t]$. Considering our present time as t, we can say $[t-\tau, t)$ is the past time interval. Thus we conclude the main feature of MDD that is the functional value in real time depends on the past time also. That is why D_τ is called the non-local operator whereas integer order derivative or integration) is a local operator (i.e. it does not depend on the past time). The kernel function $K(t-\xi)$ can be chosen freely, such as $1, [1-(t-\xi)], [1-(t-\xi)/\tau]^p$ for any positive real number p which may be more practical. They are a monotonic increasing function from 0 to 1 in the interval $[t-\tau, t]$. According to the nature of the problem, one can select a suitable kernel function of his/her choice.

4. Thermoelasticity model using MDDs

From the Maxwell–Cattaneo theory to Green–Naghdi generalized thermoelasticity models, it is well established that thermal memory has a significant role in the theory of thermoelasticity. In the twenty-first century, memory components have been introduced in terms of fractional-order derivatives in numerous forms, as follows:

(1) Sherief et al. [31] introduced fractional derivatives in the heat flux laws and modified Fourier's law in the following manner:

$$\left(1+\tau^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)q_i = -K_T\Theta_{,i}, \quad 0<\alpha\leq 1,$$

where q_i are the heat flux components, $\Theta = T - T_0$ is the temperature increment above the uniform reference temperature T_0 of the medium, T is the absolute temperature of the medium and K_T is the thermal conductivity.

(2) Youssef [32] introduced a fractional integral into the heat flow relation

$$\left(1+\tau\frac{\partial}{\partial t}\right)q_i = -K_T I^{\alpha-1}\Theta_{,i}, \quad 0<\alpha\leq 2,$$

where $I^{(\cdot)}$ is the Riemann–Liouville integral operator.

(3) Ezzat and Fayik [33] adopted the generalized fractional-order Taylor series expansion

$$\left(1+\frac{\tau^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)q_i = -K_T\Theta_{,i}, \quad 0<\alpha\leq 1.$$

In these fractional models of modified heat flux laws, the memory response is described by the fractional index parameter.

(4) Yu et al. [34] introduced MDDs in the heat conduction law in the following way:

$$(1+\tau\mathcal{D}_a)\,q_i = -K_T\Theta_{,i},$$

where $D_a f(t) = D_a^{(1)} f(t)$.

(5) Later, Ezzat et al. [22, 26] introduced the first order MDD, instead of fractional calculus, into the rate of heat flux in LS theory [4] of generalized thermoelasticity to denote memory-dependence as:

$$\left(1+\tau_0\mathcal{D}_{\tau_0}\right) q_i = -K_T\Theta_{,i},$$

where τ_0 is introduced as the time delay parameter. Equation (15) provides the following advantages compared with the aforementioned amendments of Fourier's law by using fractional derivatives: (1) the influence of memory dependency claims it's superiority in terms of memory scale parameter; (2) in a limiting sense, this simplification develops the LS model of generalized thermoelasticity; and (3) because the kernel function and the memory scale parameters may be chosen subjectively, it is more malleable in many practical applications.

5. Governing equations and formulation of the problem

In this paper, we consider a homogeneous isotropic, thermally and electrically conducting elastic half-space: $M = \{(x, y, z); -\infty < x, y < \infty,\ 0 \leq z < \infty\}$, at uniform reference temperature T_0, in the undisturbed state and discuss the thermal and elastic plane waves motion of small amplitude. The medium M is under the action of a uniform magnetic field of intensity $\vec{H}_0$ acting in the positive direction of y-axis, so that $\vec{H}_0 = (0, H_0, 0)$ (H_0 is a constant). Let the origin O of a fixed rectangular Cartesian coordinate system $Oxyz$ be fixed at a point on the plane boundary $z = 0$ with z-axis pointing vertically downward into M and x-axis is directed along the horizontal direction (see Figure 1). The y-axis is taken in the direction of the line of intersection of the plane wave front with the plane surface. The boundary surface $z = 0$ is assumed to be thermally insulated and free from mechanical stresses. Due to the application of the magnetic field $\vec{H}_0$, an induced magnetic field $\vec{h} = (0, h, 0)$, an induced electric field $\vec{E} = (E_1, 0, E_3)$, and an electric current density $\vec{J} = (J_1, 0, J_3)$ are developed in the medium M which satisfy the following simplified linearized equations of electrodynamics of slowly moving continuous media having perfect electrical conductivity in absence of displacement current [12]:

$$\vec{J} = \vec{\nabla} \times \vec{h}, \tag{3}$$

$$\vec{\nabla} \times \vec{E} = -\mu_0\dot{\vec{h}}, \tag{4}$$

$$\vec{E} = -\mu_0(\dot{\vec{u}} \times \vec{H}), \tag{5}$$

$$\nabla \cdot \vec{h} = 0, \tag{6}$$

where $\vec{J}$ is the electric current density, $\vec{H}$ is the total magnetic field, μ_0 is the magnetic permeability of solid, and $\vec{H} = (0, H_0 + h, 0)$ is the total magnetic field. The small effect of temperature gradient on $\vec{J}$ is ignored. We also assume that both $\vec{h}$ and $\vec{E}$ are small in magnitude in accordance with the assumptions of the linear theory of thermoelasticity. Also, there arises the Lorentz Force, $\vec{F} = \mu_0(\vec{J} \times \vec{H})$. Due to the effect of the force, points of the medium undergo a displacement vector $\vec{u}$, which gives rise to a temperature. Motivated by this fact, the displacement-temperature formulation is adopted here although in some other practical cases the stress-temperature formulations have a number of advantages.

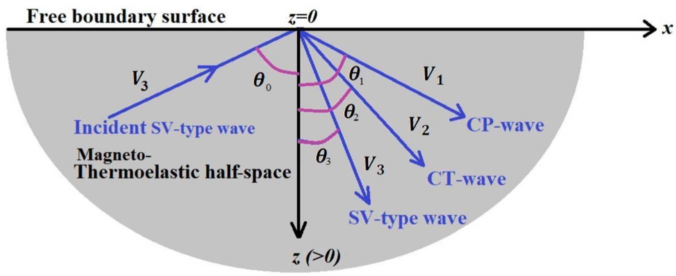

Figure 1. Geometry of the problem showing incident and reflected waves at the surface $z = 0$.

If we restrict our analysis to a plane strain parallel to xz−plane with displacement vector $\vec{u}(x, z, t) = (u, 0, w)$ and temperature change $\Theta(x, z, t)$, then the fundamental equations of motion, heat conduction equation and the stress-strain-temperature relation, in absence of heat sources, in generalized thermoelasticity with memory-dependent heat transfer developed by Ezzat et al. [22, 26] can be written in vector form as

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\vec{\nabla}\vec{\nabla}\cdot\vec{u} - \gamma\vec{\nabla}\Theta + \vec{F} = \rho\ddot{\vec{u}}, \tag{7}$$

$$K_T\nabla^2\Theta = \frac{\partial}{\partial t}(1 + \tau\mathcal{D}_\tau)\left(\rho C_E\Theta + \gamma T_0\vec{\nabla}\cdot\vec{u}\right), \tag{8}$$

$$\vec{\tau} = \lambda\vec{\nabla}\cdot\vec{u}\vec{I} + \mu\left[\nabla\vec{u} + \nabla\vec{u}^T\right] - \gamma\Theta\vec{I}, \tag{9}$$

where $\nabla^2 \equiv (\partial^2/\partial x^2 + \partial^2/\partial z^2)$, λ, μ are Lamé constants, $\gamma = (3\lambda + 2\mu)\alpha_T$ is the thermoelastic coupling parameter, α_T is the coefficient of linear thermal expansion, ρ is the mass density, C_E is the specific heat at constant strain, $\vec{\tau}$ is the stress tensor, and $\vec{I}$ is the identity tensor of order 3. All the considered functions are assumed to be bounded as $x \to +\infty$. The comma notation is used for spatial derivatives and the superposed dot represents the time differentiation.

Equations (3)–(5) yield the Lorentz Force vector, $\vec{F}$ as

$$\vec{F} = \mu_0 H_0^2\left(\vec{\nabla}\vec{\nabla}\cdot\vec{u}\right). \tag{10}$$

In our present study, we shall deal with the following kernel function:

$$K(t-\xi) = A + B(t-\xi) = \begin{cases} \frac{1}{2} & \text{if} \quad A = \frac{1}{2},\ B = 0, \\ \frac{1}{2} - \left(\frac{t-\xi}{\tau}\right) & \text{if} \quad A = \frac{1}{2},\ B = -\frac{1}{\tau}, \\ 1 - (t-\xi) & \text{if} \quad A = 1,\ B = -1, \end{cases} \tag{11}$$

where A and B are constants.

Equations (7) and (8) represent a fully hyperbolic system that permits finite speed for both elastic and thermal disturbances. Following Ezzat and Youssef [35], we now define the following dimensionless quantities

$$(x', z') = C_L\eta(x, z),\ (u', w') = C_L\eta(u, w),\ t' = C_L^2\eta t\,, \Theta' = \frac{\gamma\Theta}{\rho C_L^2},\ \tau'_{ij} = \frac{\tau_{ij}}{\rho C_L^2},$$

where $C_L^2 = (\lambda + 2\mu)/\rho$ is the speed of classical longitudinal wave (P-wave) and $\eta = \rho C_E/K_T$.

Upon using the above quantities along with Equation (10) into Equations (7)–(9), and suppressing the primes for convenience, we obtain

$$\beta^2\nabla^2\vec{u} + (1 - \beta^2 + R_M)\vec{\nabla}\vec{\nabla}\cdot\vec{u} - \vec{\nabla}\Theta = \ddot{\vec{u}}, \tag{12}$$

$$\nabla^2\Theta = \frac{\partial}{\partial t}(1 + \tau\mathcal{D}_\tau)\left(\Theta + \varepsilon_T\vec{\nabla}\cdot\vec{u}\right), \tag{13}$$

$$\vec{\tau} = (1 - 2\beta^2)\vec{\nabla}\cdot\vec{u}\vec{I} + \beta^2\left[\nabla\vec{u} + \nabla\vec{u}^T\right] - \Theta\vec{I}, \tag{14}$$

where $\beta^2 = \mu/(\lambda + 2\mu)$ is the ratio of the classical SV-wave speed to the classical P-wave speed, $R_M = \mu_0 H_0^2/(\lambda + 2\mu)$ is the magnetic pressure number [9], and $\varepsilon_T = \gamma^2 T_0/[\rho C_E(\lambda + 2\mu)]$ is the dimensionless thermoelastic coupling constant.

We now introduce the displacement potentials ϕ (corresponds to dilatational wave) and ψ (corresponds to shear wave) through the Helmholtz vector representation as

$$u = \vec{\nabla}\phi + \vec{\nabla}\times\vec{\psi}, \quad \vec{\nabla}\cdot\vec{\psi} = 0. \tag{15}$$

Inserting (15) into Equations (12)–(14), we write

$$\vec{\nabla}\left[\beta^2\nabla^2\phi + (1 - \beta^2 + R_M)\nabla^2\phi - \ddot{\phi} - \Theta\right] + \vec{\nabla}\times\left[\beta^2\nabla^2\vec{\psi} - \ddot{\vec{\psi}}\right] = 0, \tag{16}$$

$$\nabla^2\Theta - (1 + \tau D_\tau)\,\dot{\Theta} = \varepsilon_T\,(1 + \tau D_\tau)\,\nabla^2\dot{\phi}. \tag{17}$$

Equation (16) will be satisfied if

$$(1 + R_M)\nabla^2\phi - \frac{\partial^2\phi}{\partial t^2} - \Theta = 0, \tag{18}$$

$$\beta^2\nabla^2\vec{\psi} - \frac{\partial^2\vec{\psi}}{\partial t^2} = 0. \tag{19}$$

Equations (17) and (18) reveal that the thermal field Θ is coupled with the potential ϕ only and so creates two coupled longitudinal waves, namely, a coupled dilatational elastic wave (CP-wave) and a coupled thermal wave (CT-wave). Eliminating Θ between Equations (17) and (18), we get

$$(1 + R_M)\nabla^4\phi - \left[(1 + \varepsilon_T + R_M)\,(1 + \tau D_\tau)\,\nabla^2\dot{\phi} + \nabla^2\ddot{\phi}\right] + (1 + \tau D_\tau)\,\dddot{\phi} = 0. \tag{20}$$

Considering (15), we may choose $\vec{\psi} = (0, \psi, 0)$, and hence Equation (19) takes the form

$$\nabla^2\psi = \frac{1}{\beta^2}\ddot{\psi}. \tag{21}$$

Thus, the potential ψ corresponds to the displacement motion in the *xz*-plane due to a SV-type wave.

6. Dispersion equation and its solutions

For a harmonic plane wave propagating in the direction where the wave normal vector lies in the xz−plane making an angle θ_0 with the positive z-axis, the solutions of Equations (20) and (21) may be assumed as

$$(\phi, \psi) = \left(\phi^0, \psi^0\right) \exp\{\iota k(x \sin\theta_0 - z\cos\theta_0) - \iota\omega t\}, \tag{22}$$

where ϕ^0, ψ^0 are the constants (possibly complex) representing the coefficients of the wave amplitudes, $\iota = \sqrt{-1}$, k is the dimensionless wavenumber (possibly complex) to be determined, and ω is the dimensionless assigned real angular frequency. If we set $k = \Re(k) + \iota\Im(k)$, (where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of a complex number, respectively), it may be verified that for the waves to be physically realistic, we should have $\Re(k) > 0$ and $\Im(k) \geq 0$ and that only the real parts of the solution (22) are physically relevant [36]. Also note that x and z in (22) are both non-negative, i.e. $x, z \geq 0$. Then, on the surface $z = 0$, the term $\exp[-x\Im(k)\sin\theta_0] \to 0$ as $x \to +\infty$ which in turn means that the energy represented by the wave solutions (22) is bounded (Billingham and King [37]). Further, the solution (22) corresponds to waves for which $\omega/\Re(k)$ is the phase speed and $\Im(k)$ is the decay (attenuation) coefficient.

Substituting from Equation (22) into Equations (20) and (21), we get

$$k^4 - L_1 k^2 + L_2 = 0, \tag{23}$$

$$k^2 = \frac{\omega^2}{\beta^2}, \tag{24}$$

where

$$L_1 = \frac{\iota\omega(1+G)(1+\varepsilon_T+R_M)+\omega^2}{(1+R_M)},\ L_2 = \frac{i\omega^3(1+G)}{(1+R_M)},\ G \equiv G(\tau,\omega)$$
$$= \frac{(A\omega+\iota B)[1-\exp(\iota\tau\omega)]}{\omega} - B\tau\exp(\iota\tau\omega). \tag{25}$$

The quadratic Equation (23) in k^2 is the general dispersion relation for the wave propagation in generalized thermoelasticity with MDD. Clearly, the coefficients L_1 and L_2 in (25) are complex for $\omega > 0$. The two roots of (23) and the only root of Equation (24) are given by

$$k_{2,1}^2 = \frac{1}{2}\left[L_1 \pm \sqrt{L_1^2 - 4L_2}\right], \tag{26}$$

$$k_3^2 = \frac{\omega^2}{\beta^2}. \tag{27}$$

Here k_1^2 corresponds to " −" sign and k_2^2 corresponds to "+" sign. For a given positive ω, Equation (23) gives us four roots of the form $\pm k_1$ and $\pm k_2$, for k. Of these four roots, only two roots yield positive values for $\Re(k)$ with $\Im(k_{1,2}) \geq 0$. Hence, there are two distinct traveling coupled longitudinal waves of wave number $k_{1,2}$, namely, a coupled dilatational elastic wave (CP-wave) and a coupled thermal wave (CT-wave). Both of these waves are influenced by elastic as well as thermal and magnetic fields. This result agrees with that in the conventional coupled as well as LS theories [2, 4] of thermoelasticity. In these theories, it has also been observed that one of the longitudinal waves is a CP-wave and the other is a CT-wave. The phase speeds of the CP- and CT-waves are given by $V_{1,2} = \omega/\Re(k_{1,2})$. Since the attenuation coefficients $\Im(k_{1,2})$ and phase speeds $V_{1,2}$ are functions of ω, which in turn means that the coupled dilatational elastic-thermal waves suffer attenuation and dispersion due to the magneto-thermoelastic character of the considered medium. The magnetic field, elastic material properties of the medium, time-delay and kernel function of MDD influence the dispersion and attenuation nature of the waves. Besides, since the wavenumbers of both the waves are complex, so they are inhomogeneous waves.

In case of UCT ($\varepsilon_T = 0$) without considering the magnetic field effect ($H_0 = 0$), we find

$$V_1(\varepsilon_T = 0) = 1, \quad V_2(\varepsilon_T = 0) = \frac{\sqrt{\omega}}{\Re[\iota(1+G)]^{1/2}}.$$

Thus for the present problem, we conclude that while V_1 represents the speed of CP-wave, V_2 the speed of CT-wave (according to our consideration of the sign of k_1^2 and k_2^2). For $\varepsilon_T \neq 0$, the CP-wave and CT-wave are coupled dilatational elastic-thermal waves and the coupling is measured by the following amplitudes ratio:

$$\zeta_j = \left[\omega^2 - (1+R_M)k_j^2\right] = \frac{\varepsilon_T \omega (1+G) k_j^2}{\left[\omega(1+G) + \iota k_j^2\right]} \quad (j = 1,2). \tag{28}$$

A look at Equation (27) reveals that there exist one SV-type wave of wavenumber k_3 whose phase speed is

$$V_3 = \beta. \tag{29}$$

Expression (29) indicates that the SV-type wave remains unaffected by the presence of the magnetic field and thermal wave effect. It is also noted that this wave is non-dispersive and propagates in the medium considered without being attenuated.

7. Perturbation solution of dispersive waves

The perturbation method has been widely used [10–12, 14, 38] to study the wave propagation problems in classical (coupled) and non-classical (generalized) thermoelastic continua. Here, our aim is to derive the perturbation solutions of the instant problem in this section. The dispersion Equation (23) can also be rewritten as

$$f(k^2) - \varepsilon_T g(k^2) = 0, \tag{30}$$

where

$$f(k^2) = (1+R_M)k^4 - k^2\left[\iota\omega(1+G)(1+R_M) + \omega^2\right] + i\omega^3(1+G),$$
$$g(k^2) = \iota\omega(1+G)k^2. \tag{31}$$

For most of the materials the thermo-mechanical coupling parameter ε_θ is very small and therefore, we develop series expansions in terms of ε_θ for the roots k_j^2 $(j = 1,2)$ of Equation (23) in order to explore the effect of various interacting fields on the waves.

Thus, for $\varepsilon_T \ll 1$, we obtain the the perturbation solutions for the roots from (30) and (31) as

$$k_1^2(\varepsilon_T) = \frac{\omega^2}{(1+R_M)}\left[1 - \frac{(1+G)}{(1+G)(1+R_M)+\iota\omega}\varepsilon_T + \cdots\right], \tag{32}$$

$$k_2^2(\varepsilon_T) = \iota\omega(1+G)\left[1 + \frac{(1+G)}{(1+G)(1+R_M)+\iota\omega}\varepsilon_T + \cdots\right]. \tag{33}$$

8. Reflection phenomena of magneto-thermoelastic waves

Let a train of SV-type wave having amplitude B_0 and phase speed V_3 is made incident making an angle θ_0 with the normal to the free surface $z=0$ as shown in Figure 1. Assuming that the radiation in vacuum is neglected, when SV-type wave impinges the boundary $z=0$, three reflected waves in the medium are created. Suppose the reflected CP-, CT- and SV-type waves make angles θ_1, θ_2 and θ_3 respectively with positive the z-axis. Then the complete structure of the wave fields consisting of the incident and reflected waves in the medium M may be written as

$$\phi = A_1 \exp\{\iota k_1(x\sin\theta_1 + z\cos\theta_1) - \iota\omega t\} + A_2 \exp\{\iota k_2(x\sin\theta_2 + z\cos\theta_2) - \iota\omega t\}, \tag{34}$$

$$\Theta = \zeta_1 A_1 \exp\{\iota k_1(x\sin\theta_1 + z\cos\theta_1) - \iota\omega t\} + \zeta_2 A_2 \exp\{\iota k_2(x\sin\theta_2 + z\cos\theta_2) - \iota\omega t\}, \tag{35}$$

$$\psi = B_0 \exp\{\iota k_3(x\sin\theta_0 - z\cos\theta_0) - \iota\omega t\} + B_1 \exp\{\iota k_3(x\sin\theta_3 + z\cos\theta_3) - \iota\omega t\}, \tag{36}$$

where A_1, A_2 and B_1 represent the coefficients of amplitudes of the reflected CP-, CT- and SV-type waves, respectively. The reflection coefficient is defined as the amplitude ratio of the reflected wave to the incident wave and is determined by the appropriate boundary conditions on the surface $z=0$.

We consider the surface $z=0$ as stress-free and thermally insulated. These conditions can be written mathematically as:

$$\tau_{zz} + \bar{\tau}_{zz} = \tau_{xz} + \bar{\tau}_{xz} = \frac{\partial\Theta}{\partial z} = 0, \quad \text{at } z=0, \tag{37}$$

where $\bar{\tau}_{ij}$ is the Maxwell's electro-magneto stress tensor, given by

$$\bar{\tau}_{ij} = \mu_0\left[H_i h_j + H_j h_i - (\vec{H}\cdot\vec{h})\delta_{ij}\right] \quad (i,j = x,z). \tag{38}$$

In terms of displacement potential functions, first two conditions in (37) are simplified to

$$(1+R_M)\left(\frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\phi}{\partial x^2}\right) + 2\beta^2\left(\frac{\partial^2\psi}{\partial x\partial z} - \frac{\partial^2\phi}{\partial x^2}\right) - \Theta = 0, \tag{39}$$

$$\left(2\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right) = 0, \quad \text{at } z=0. \tag{40}$$

In order to satisfy the above boundary conditions at $z=0$, the following relation must be hold on $z=0$:

$$k_3\sin\theta_0 = k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3, \tag{41}$$

or in the form

$$\theta_0=\theta_3 \quad \text{and} \quad \frac{\sin\theta_1}{V_1}=\frac{\sin\theta_2}{V_2}=\frac{\sin\theta_0}{V_3}, \quad \text{at } z=0, \tag{42}$$

which is often refereed as *extended Snell's law*.

Substituting from Equations (34)–(36) into the boundary conditions (37), (39) and (40), and using the relation (41), the following system of equations for the reflection coefficients $R_{SP}=A_1/A_0$, $R_{ST}=A_2/A_0$, $R_{SS}=B_1/A_0$ is obtained:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 0 \end{bmatrix}\begin{bmatrix} R_{SP} \\ R_{ST} \\ R_{SS} \end{bmatrix}=\begin{bmatrix} -a_{13} \\ a_{23} \\ 0 \end{bmatrix}, \tag{43}$$

where

$$a_{11}=\omega^2-2\beta^2k_1^2\sin^2\theta_0,\ a_{12}=\omega^2-2\beta^2k_2^2\sin^2\theta_2,\ a_{13}=\omega^2\sin 2\theta_3,$$
$$a_{21}=k_1^2\sin 2\theta_0,\ a_{22}=k_2^2\sin 2\theta_2,\ a_{23}=-k_3^2\cos 2\theta_3,$$
$$a_{31}=\zeta_1k_1\cos\theta_1,\ a_{32}=\zeta_2k_2\cos\theta_2.$$

After solving (43), we get the reflection coefficients in explicit forms as follows:

$$R_{SP}=\frac{\beta^2\zeta_2k_2k_3^2\sin(4\theta_0)\cos(\theta_2)}{\begin{array}{c}-\zeta_1k_1\cos(\theta_1)\left(\omega^2\cos(2\theta_0)+2\beta^2k_2^2\sin(2\theta_0-\theta_2)\sin(\theta_2)\right)+\\ 2\beta^2\zeta_2k_1^2k_2\sin(2\theta_0-\theta_1)\sin(\theta_1)\cos(\theta_2)+\zeta_2k_2\omega^2\cos(2\theta_0)\cos(\theta_2)\end{array}}, \tag{44}$$

$$R_{ST}=\frac{\beta^2\zeta_1k_1k_3^2\sin(4\theta_0)\cos(\theta_1)}{\begin{array}{c}\zeta_1k_1\cos(\theta_1)\left(\omega^2\cos(2\theta_0)+2\beta^2k_2^2\sin(2\theta_0-\theta_2)\sin(\theta_2)\right)-\\ 2\beta^2\zeta_2k_1^2k_2\sin(2\theta_0-\theta_1)\sin(\theta_1)\cos(\theta_2)+\zeta_2k_2\omega^2(-\cos(2\theta_0))\cos(\theta_2)\end{array}}, \tag{45}$$

$$R_{SS}=\frac{\begin{array}{c}\zeta_1k_1\cos(\theta_1)\left(2\beta^2k_2^2\sin(\theta_2)\sin(2\theta_0+\theta_2)-\omega^2\cos(2\theta_0)\right)-\\ 2\beta^2\zeta_2k_1^2k_2\sin(\theta_1)\sin(2\theta_0+\theta_1)\cos(\theta_2)+\zeta_2k_2\omega^2\cos(2\theta_0)\cos(\theta_2)\end{array}}{\begin{array}{c}\zeta_1k_1\cos(\theta_1)\left(\omega^2\cos(2\theta_0)+2\beta^2k_2^2\sin(2\theta_0-\theta_2)\sin(\theta_2)\right)-\\ 2\beta^2\zeta_2k_1^2k_2\sin(2\theta_0-\theta_1)\sin(\theta_1)\cos(\theta_2)+\zeta_2k_2\omega^2(-\cos(2\theta_0))\cos(\theta_2)\end{array}}. \tag{46}$$

It is observed that these reflection coefficients depend on the angle of incidence (θ_0), magnetic pressure number R_M and the material properties of the thermoelastic medium. It can be noted that for uncoupled thermoelasticity ($\varepsilon_T=0$), $\zeta_j=0$ ($j=1,2$) and hence there is no reflected CT-wave. So, in this case $R_{ST}=0$ at all angle of incidence θ_0.

9. Total reflection

We now consider the most interesting case namely, the case of total reflection beyond the critical angle. By using the relation (41) into the solution (34), we get (after omitting the factor $\exp(-\iota\omega t)$)

$$\phi = A_1 \exp\{\iota k_1(x\sin\theta_1 + z\cos\theta_1)\} + A_2 \exp\{\iota k_2(x\sin\theta_2 + z\cos\theta_2)\},$$
$$= \exp\left\{\frac{\iota\omega}{V_3}x\sin\theta_0\right\}\left[\bar{A}_1 \exp\left\{\frac{\iota\omega z}{V_3}\sqrt{\frac{V_3^2}{V_1^2} - \sin^2\theta_0}\right\} + \bar{A}_2 \exp\left\{\frac{\iota\omega z}{V_3}\sqrt{\frac{V_3^2}{V_2^2} - \sin^2\theta_0}\right\}\right], \tag{47}$$

where $\bar{A}_j = A_j \exp\{-\Im(k_j)(x\sin\theta_j + z\cos\theta_j)\}$, $\Im(k_j) \geq 0$ $(j = 1, 2)$.

Since $V_1 < V_2$, hence $V_3/V_2 < V_3/V_1$ and when θ_0 increases, $\sin(\theta_0)$ increases to the value V_3/V_2 first. If $\sin\theta_0 = V_3/V_2 = \sin\theta_C$, then $\theta_0 = \theta_C$ is called the critical angle. For $\theta_0 > \theta_C$, the factor $\sqrt{V_3^2/V_1^2 - \sin^2\theta_0}$ becomes purely imaginary. The critical angle θ_C for the elastic case is given by $\theta_C = \sin^{-1}(\beta)$ [39]. When $\sin\theta_0 > V_3/V_2$, we obtain from (47) that

$$\phi = \exp\left\{\frac{\iota\omega}{V_3}x\sin\theta_0\right\}\left[\bar{A}_1 \exp\left\{-\frac{\omega z}{V_3}\sqrt{\sin^2\theta_0 - \frac{V_3^2}{V_1^2}}\right\} + \bar{A}_2 \exp\left\{-\frac{\omega z}{V_3}\sqrt{\sin^2\theta_0 - \frac{V_3^2}{V_1^2}}\right\}\right], \tag{48}$$

Thus the thermal part and elastic part of the reflected coupled dilatational elastic-thermal wave propagates horizontally in the x-direction whereas its amplitude decays exponentially with depth (z).

10. Numerical results and discussions

In this section, we perform some numerical calculations in order to illustrate the analytical results. For this purpose, we choose copper like material whose physical data are [26]: $\lambda = 7.76 \times 10^{10}$ N/m^2, $\mu = 3.86 \times 10^{10}$ N/m^2, $T_0 = 293$ K, $\rho = 8954$ kg/m^3, $C_e = 383.1$ m^2/K, $K_T = 386$ N/K s, $\alpha_T = 383.1$ K^{-1}, $\varepsilon_T = 0.0168$, $R_M = 0.5$.

For the purpose of the numerical computations, we have selected a fixed kernel, namely $K(t,\xi) = 1/2 - (t-\xi)/\tau$. Using the above values of the material constants, the critical angle is obtained as $\theta_C = 20.6^\circ$.

Figure 2 expresses a comparison between the reflection coefficients $|R_{SP}|$, $|R_{ST}|$ and $|R_{SS}|$ within the range of the incident angle θ_0 ($0^\circ \leq \theta_0 \leq 90^\circ$) of the incident SV-type wave. It is noticed that the reflection coefficient of the reflected CP-wave dominates while the reflected CT- and SV-type waves are both smaller than the reflection coefficient of the CP-wave. The maximum value of $|R_{SP}|$ occurs at $\theta_0 = 0^\circ$, while that of $|R_{ST}|$ and $|R_{SS}|$ occur at the critical angle $\theta_C = 20.60^\circ$. The reflection coefficient of the SV-type wave attains its minimum value, 0 at $\theta_0 = 0^\circ$.

The impact of the magnetic pressure number parameter R_M upon the variations of the moduli of the reflection coefficients are important in application point of view. It is evident from Figure 3 that all the reflection coefficients show significant changes for different values of R_M. The reflection coefficients $|R_{ST}|$ and $|R_{SS}|$ decrease with an increase in the value of R_M. On the contrary, the reflection coefficient $|R_{SP}|$ increases with an increase in R_M. The pattern of the curves in each of the reflection coefficient are similar apart from the magnitudes.

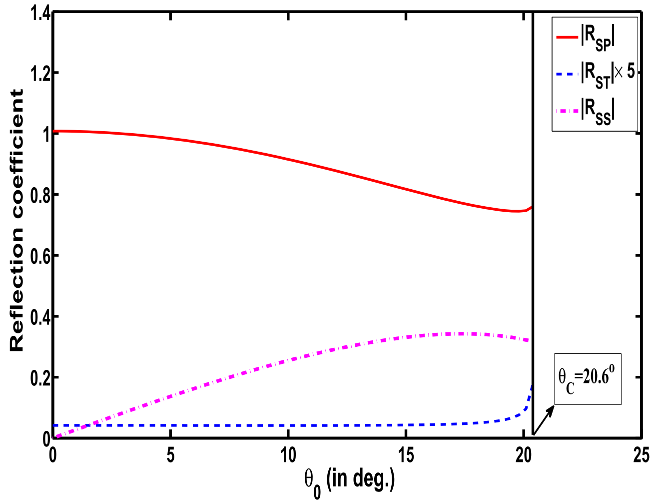

Figure 2. Comparison of the reflection coefficients with respect to θ_0.

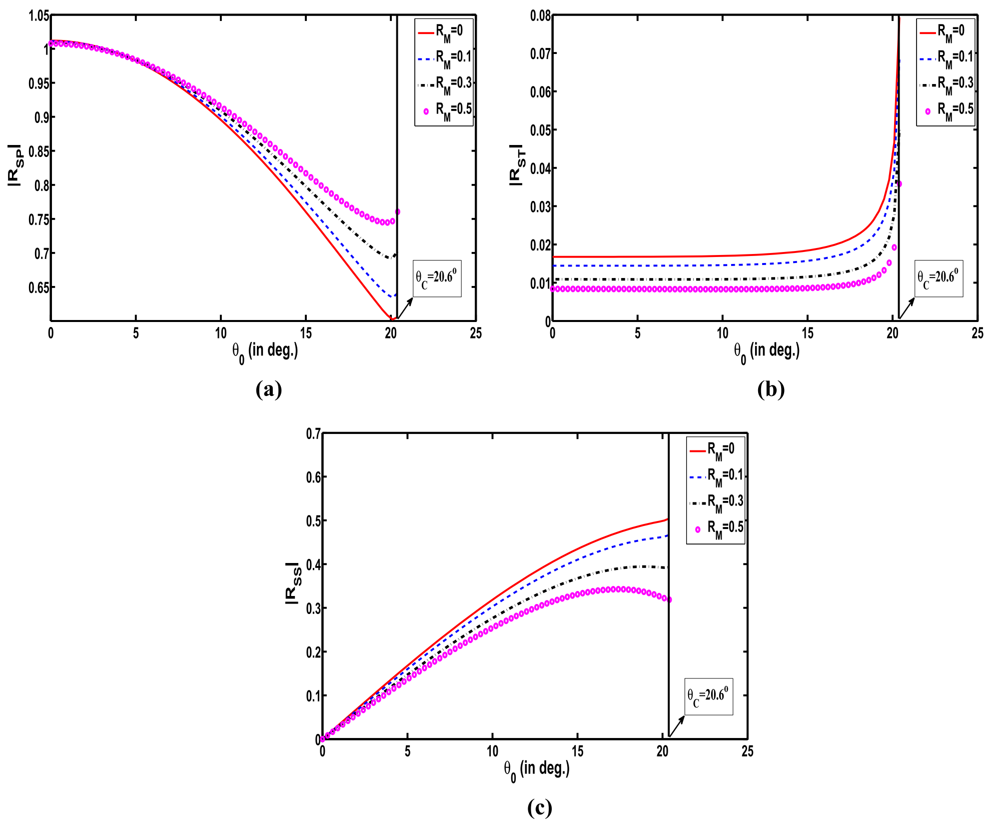

Figure 3. Effect of the magnetic pressure number (R_M) on the reflection coefficients.

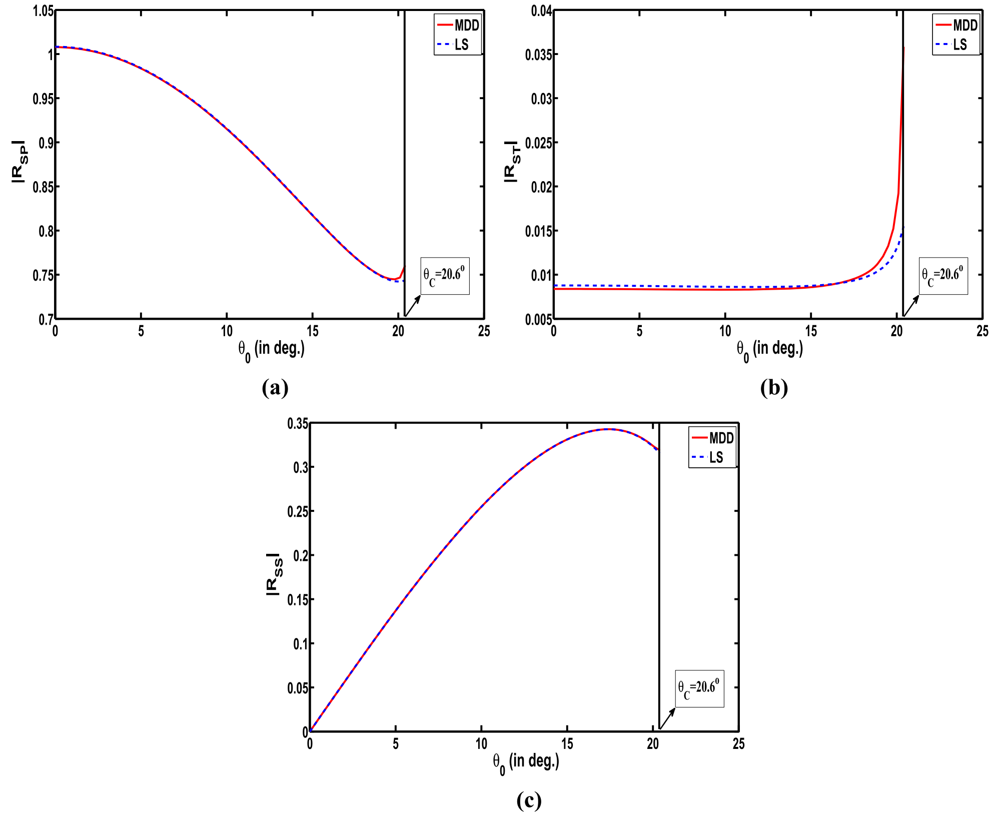

Figure 4. Comparison of the reflection coefficients for the MDD and LS models.

Figure 4 shows a comparison between the reflection coefficients obtained for the MDD and the LS theories. It is evident from the Figures 4(a, c) that the reflection coefficients $|R_{SP}|$ and $|R_{SS}|$ coincide for both the theories within the whole range of θ_0 in contrast to the reflection coefficient $|R_{ST}|$. The difference in the profile of $|R_{ST}|$ is occurred due to the presence of the MDD in the heat conduction equation.

The influence of the Poison ratio σ upon the variations of the magnitudes of the reflection coefficients are also interesting. It is evident from Figure 5 that the reflection coefficient of the CP-wave is most sensitive to the Poison ratio while the reflection coefficient of the CT-wave is most insensitive to σ. The increasing of the Poison ratio makes the reflected CT-wave weaken. The reflection coefficients $|R_{SP}|$ and $|R_{ST}|$ increase with an increase in the value of σ in contrast to the reflection coefficient $|R_{SS}|$.

Figure 6 is drawn to show the effect of the thermo-mechanical coupling parameter (ε_T) on $|R_{SP}|$, $|R_{ST}|$ and $|R_{SS}|$. From Figure 6, we observe that the absolute values of R_{SP} and R_{ST} have large values for the large value of ε_T meaning it has an increasing effect on $|R_{SP}|$ and $|R_{ST}|$ at each angle of incidence. On the other hand, ε_T shows an decreasing effect on $|R_{SS}|$. We can also see from these figures that the influence of the coupling parameter on $|R_{SP}|$ and $|R_{SS}|$ is significantly pronounced whereas ε_T has a small effect on $|R_{SS}|$.

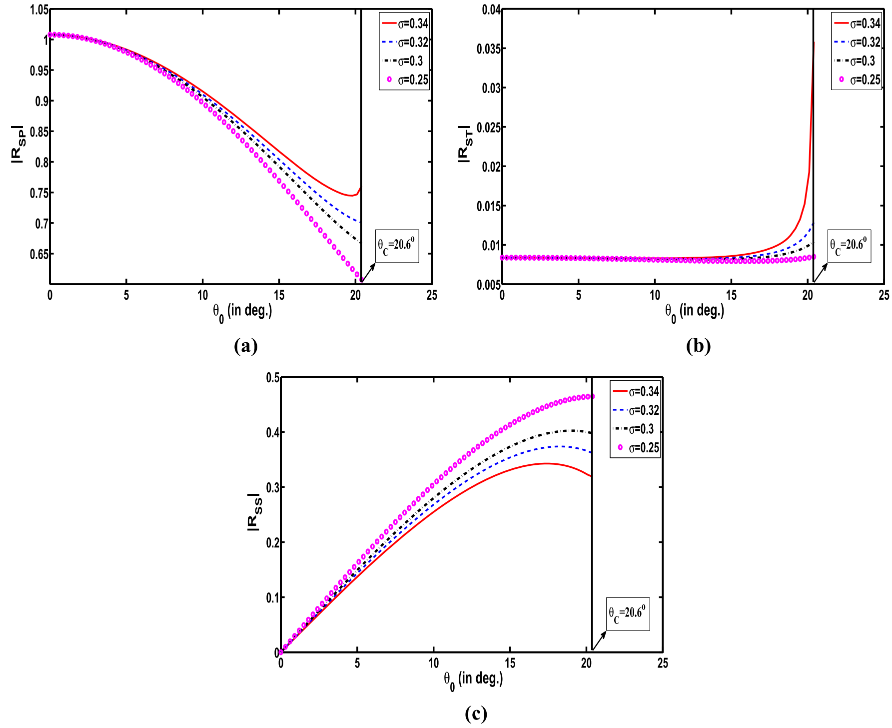

Figure 5. Variations of the reflection coefficients for different σ.

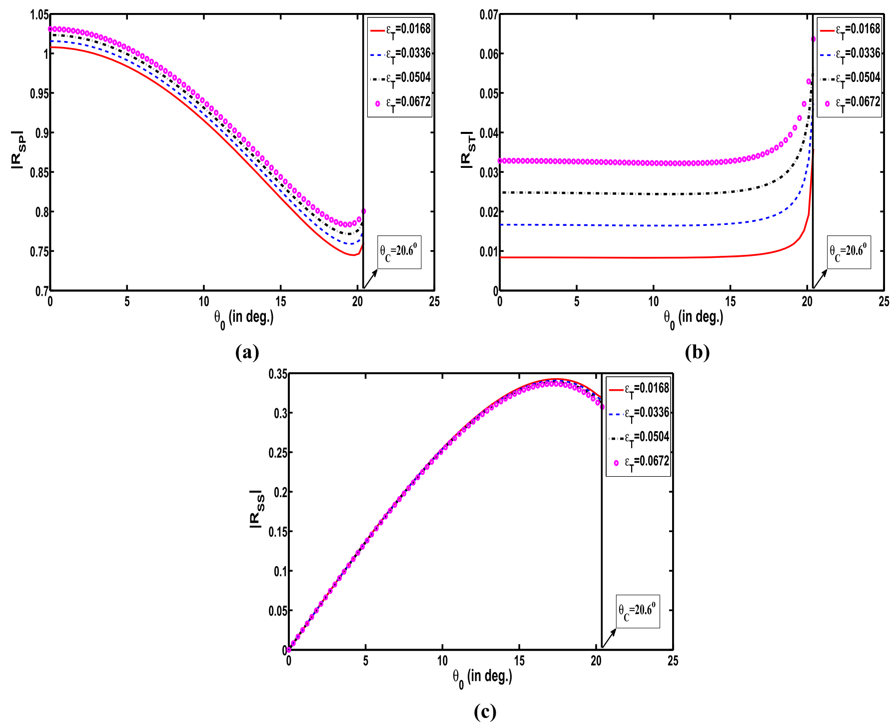

Figure 6. Variations of the reflection coefficients for different ε_T.

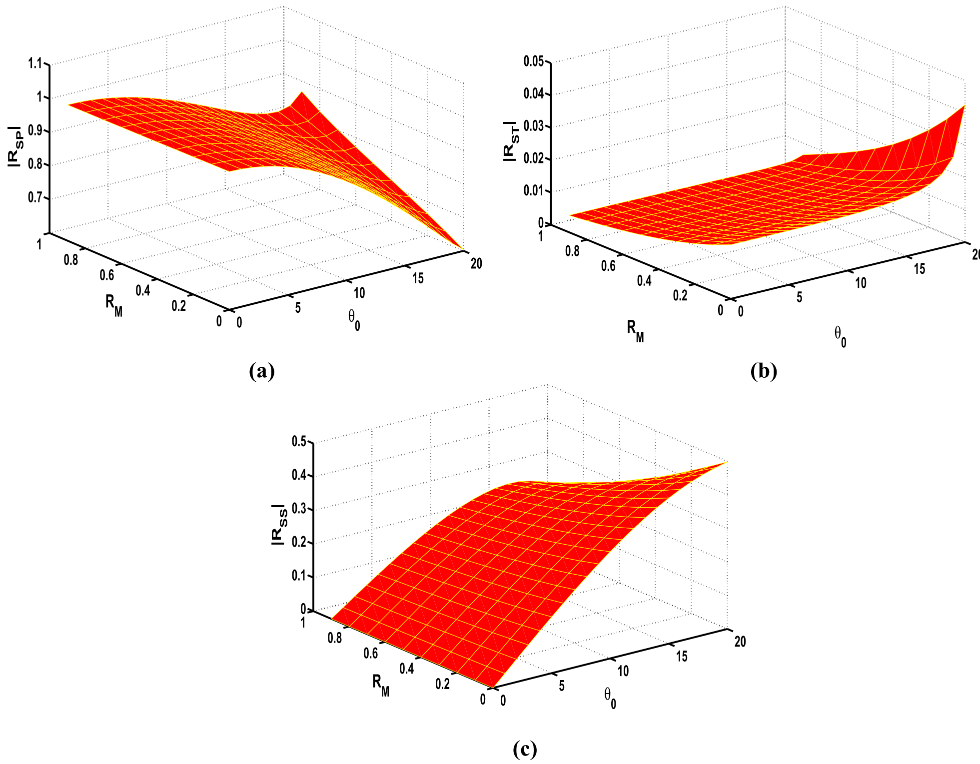

Figure 7. Three-dimensional variations of the reflection coefficients with respect to θ_0 and R_M.

In Figure 7, we reveals the three-dimensional distributions of $|R_{SP}|$, $|R_{ST}|$ and $|R_{SS}|$ with respect to the incident angle θ_0 and the magnetic pressure number R_M. Finally, we observe from Figures 2–6 that the critical angle (θ_C) of the incident SV-type wave for thermoelastic case is $\theta_C = 20.6^\circ$ which is exactly the same with the calculated value of θ_C.

In Figure 8, we plot the phase speeds and the attenuation coefficients of the CP-wave and CT-wave with the angular frequency ω. We noticed that the phase speeds and the attenuation coefficients of the CP-wave and the CT-wave have larger values for the L-S theory as compared to the MDD theory where the only exception is found when we plot the attenuation coefficient of the CP-wave. If we compare the phase speeds for both the waves, we found CP-wave has larger magnitudes. When we compare the attenuation coefficients of both the waves, CP-wave experiences less attenuation. From this set of figures, we can say that the memory effect is more sensitive for the phase speed of the CT-wave as compared to the CP-wave. But for the case of attenuation, we have seen a reverse effect of the MDD, i.e. the presence of memory plays a more significant role for the attenuation coefficient of the CP-wave. Figures 8(c, d) reveal that the CP- and the CT-waves experience attenuation. Also, the significant changes in the profiles of the phase speeds of the CP- and CT-waves indicates that these waves are dispersive in nature.

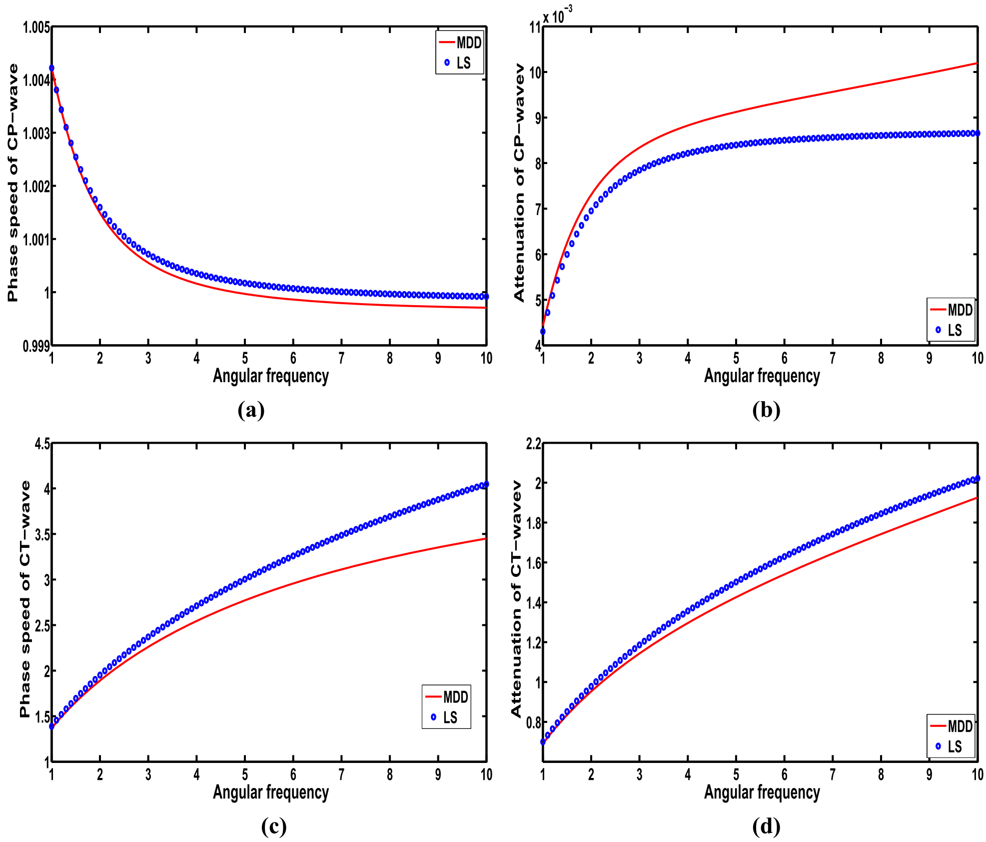

Figure 8. Comparison of the phase speeds and the attenuation coefficients of the CP-wave and CT-wave vs. non-dimensional angular frequency ω for MDD and L-S theories.

11. Conclusions

The following points can be concluded according to our present study:

(1) The reflection coefficients are functions of the angle of incidence, magnetic field and the material properties of the thermoelastic medium.
(2) Magnetic pressure number, Poison ratio and the thermoelastic coupling parameter affect significantly the reflection coefficients of all the waves. Poison ratio has no effect on the phase speeds as well as attenuation coefficients.
(3) The phase speeds of the CP- and CT-wave is larger for LS model as compared to MDD model. The CP- and CT-waves are dispersive and experience attenuation whereas the SV-type wave is non-dispersive as well as experience no attenuation as it does not depend on the angular frequency.
(4) It is observed that the MDD model supports the finite speed of thermal wave (CT-wave) propagation through the medium considered. Hence, this theory is indeed a generalized theory of thermoelasticity.

(5) The present work is of geophysical interest for investigations on earthquakes and similar phenomena in seismology and engineering where "MDD may play a significant role".

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Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Nihar Sarkar received his MSc degree in Applied Mathematics from University of Kalyani, West Bengal, India, in 2005. He is currently an Assistant Professor in Mathematics, City College, Kolkata, India. His research interests include wave propagation in elastic and thermoelastic solids.

Soumen De received his MSc and PhD degrees in Applied Mathematics from University of Calcutta, Kolkata, India in 2004 and 2009, respectively. He is now an Assistant Professor in Applied Mathematics at the Department of Applied Mathematics, University of Calcutta. His research interests include water waves and waves in continuum.

Nantu Sarkar received his MSc (Applied Mathematics) and PhD (Science) from Jadavpur University, Jadavpur, Kolkata, India in 2008 and 2013, respectively. He is currently an Assistant Professor in Applied Mathematics at the Department of Applied Mathematics, University of Calcutta, Kolkata, India. His research interests include waves in non-local elastic and thermoelastic solids.

ORCID

Soumen De http://orcid.org/0000-0001-8988-3679
Nantu Sarkar http://orcid.org/0000-0001-9144-4587

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